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Effect of electron-electron interaction on cyclotron resonance in high-mobility InAs/AlSb quantum wells

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We report observation of electron-electron (e-e) interaction effect on cyclotron resonance (CR) in InAs/AlSb quantum well heterostructures. High mobility values allow us to observe strongly pronounced triple splitting of CR line at noninteger filling factors of Landau levels \( \nu \). At magnetic fields, corresponding to \( \nu > 4 \), experimental values of CR energies are in good agreement with single-electron calculations on the basis of eight-band \( k \cdot p \) Hamiltonian. In the range of filling factors \( 3 < \nu < 4 \) pronounced, splitting of CR line, exceeding significantly the difference in single-electron CR energies, is discovered. The strength of the splitting increases when occupation of the partially filled Landau level tends to a half, being in qualitative agreement with previous prediction by MacDonald and Kallin [Phys. Rev. B 40, 5795 (1989)]. We demonstrate that such behaviour of CR modes can be quantitatively described if one takes into account both electron correlations and the mixing between conduction and valence bands in the calculations of matrix elements of e-e interaction. © 2015 AIP Publishing LLC.

I. INTRODUCTION

The vast majority of theoretical works devoted to many-body effects in two-dimensional (2D) systems utilizes a so-called “single-band” approximation with energy-independent effective mass \( m^* \) and g-factor \( g^* \). This approximation considers the given band only (for example, the conduction band), while the coupling with other bands are taken into account under the perturbation theory. As a result, the electrons are considered as the particles with quadratic dispersion that implies a 2D system has a full translational symmetry.

Such 2D systems in perpendicular magnetic field have an inherent limitation on many-body effects on cyclotron resonance (CR) energy, which is known as the Kohn’s theorem.1 According to the Kohn’s theorem, no manifestations of electron-electron (e-e) interactions can be observed unless the translational symmetry is broken either extrinsically by impurities2,3 or intrinsically by electron-phonon interaction4 or by effect of subband nonparabolicity.5,6 Since subband nonparabolicity results from the fact that periodic lattice potential in the crystal does not have continuous translational symmetry, in its presence, the Kohn’s theorem does not hold in general.

Many experimental results,9–17 performed on samples of intermediate quality, reported anomalies in the CR response without being able to discriminate between extrinsic and intrinsic effects. Experiments so far have been done mostly on GaAs-based systems. The relative intensities of observed CR peaks do not follow the ratio of the state population in Landau levels (LLs), and the peak position shows unusual dependence on temperature and magnetic fields.

To interpret such features observed in CR spectra, MacDonald and Kallin5 made the first attempt to describe effects of e-e interaction in CR. In particular, they used a generalized single mode approximation (GSMA) to describe interplay between e-e interaction and subband nonparabolicity in the range of filling factors of the LLs \( 2 \leq \nu \leq 3 \). Later, the mixing between CR modes at different \( \nu \) induced by e-e interactions, was also reported by Asano and Ando6 and Bychkov and Martinez.7,8

In above-mentioned papers5,8 subband nonparabolicity was taken into account by introducing a phenomenological constant describing the difference in CR energies. However, the matrix elements of e-e interaction were calculated by using the two-component wave functions in “single-band” approximation, while the features, arising in single-electron wave functions due to the mixing between conduction and valence bands, were totally

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II. GENERAL THEORY

A. Single-electron states

As mentioned in the Introduction, single-electron states in narrow-gap 2D systems are extensively studied by using the eight-band $k \cdot p$ Hamiltonian.\cite{18} It exactly takes into account the coupling between conduction ($\Gamma_0$) and valence ($\Gamma_7$, $\Gamma_8$) bands, as well as the influence of spin-orbit interaction and lattice-mismatch deformation on single-electron states.

We first start from the eight-band $k \cdot p$ Hamiltonian $H_{\Gamma_0,\Gamma_7,\Gamma_8}$ with the kinetic part for the conduction band in the form proposed by Foreman.\cite{14} Then we simplify $H_{\Gamma_0,\Gamma_7,\Gamma_8}$ to describe the conduction band states in $n$-type narrow-gap QWs by neglecting the terms resulting from the remote band contribution, the low-symmetry interfaces,\cite{35} and the absence of inversion center in the unit cell of bulk materials.\cite{36} In this case, the interband momentum matrix element $P$ should also be normalized to fit conduction band mass $m_c$. This procedure is described in detail elsewhere.\cite{19,21}

The similar approach was also used by Winkler\cite{37,38} to explain the experiments on CR in InAs/AlSb QWs performed by Yang et al.\cite{27} and Scriba et al.\cite{28} There was a good agreement between Winkler’s calculation and the experimental data. Previously, we have employed this approximation for a quantitative interpretation of our previous experimental results on CR (Refs. 25 and 33) and magnetotransport\cite{39-42} in InAs/AlSb QWs as well. Moreover, it can be shown that our approximation for the single-electron states can be reduced to the model proposed by Pfeffer and Zawadzki\cite{43} for the spin splitting in the conduction band in $n$-type QWs based on narrow-gap materials.

Including in single-electron Hamiltonian $H_{\{|\zeta\rangle\}$, both the electric field of ionized donors $V_{\text{donors}}(z)$ and the local part of the Coulomb energy (Hartree potential) $\phi_{e-e}$,\cite{20,21,23} in given basis of the Bloch functions at $\Gamma$-point of the Brillouin zone, the eight-component wave function $\Psi_{\{\zeta\},n,k}(\vec{r},z)$ of single-electron state is written as follows:\cite{20,21,23}

$$
\Psi_{\{\zeta\},n,k}(x,y,z) = \begin{pmatrix}
  c_1^{(i)}(z,n)|n,\tilde{k}\rangle \\
  c_2^{(i)}(z,n)|n+1,\tilde{k}\rangle \\
  c_3^{(i)}(z,n)|n-1,\tilde{k}\rangle \\
  c_4^{(i)}(z,n)|n,\tilde{k}\rangle \\
  c_5^{(i)}(z,n)|n+1,\tilde{k}\rangle \\
  c_6^{(i)}(z,n)|n+2,\tilde{k}\rangle \\
  c_7^{(i)}(z,n)|n,\tilde{k}\rangle \\
  c_8^{(i)}(z,n)|n+1,\tilde{k}\rangle 
\end{pmatrix},
$$

where $|n,\tilde{k}\rangle$ is the normalized harmonic oscillator functions,\cite{20} $n$ is the LL index, $\tilde{r}$ is the “spin” index, and $\tilde{k}$ is the parameter for the degenerate states within the same LL in the Landau gauge.

For $n = -1$, there is only one four-component wave function, while at $n = 0$, the solutions of a single-electron Schrödinger equation are two eigenfunctions.
the one that corresponds to the lower single-electron energy is labeled by index $a$, that for the higher energy is marked by $b$ (see, e.g., see Fig. 2 in Ref. 19 and Fig. 1 in Ref. 21).

### B. Excitonic representation

The resonant absorption of an electromagnetic radiation produces zero-momentum magnetic excitons, which correspond to magnetoplasmon and spin-wave excitations. The spectrum of magnetic excitons can be readily calculated by introducing the exciton creation operator:

$$A_{n,n',i,j}(\vec{k}) = \sum_p e^{i\vec{k}\cdot\vec{p} + it_0/2} a_{n,p,i}^+ a_{n',p+i,j},$$

where $a_{n,p,i}^+$, $a_{n,p,i}$ are the fermion creation and annihilation operators and $d_B$ is the magnetic length.

The energy of magnetic exciton $E_{n,n',i,j}(\vec{k})$ with respect to the energy of the ground state $|0\rangle$ is found from equation

$$\det(E_n^{(i,j)}(\vec{k}) - E_{n',n_i,n_j}(\vec{k})) = -E_{n,n',i,j}(\vec{k}) \delta_{n,n'}, \delta_{i,i'}, \delta_{j,j'} = 0,$$

where $E_{n}^{(i,j)}$ is the corresponding LL energy characterized by the “spin” $i$ and LL $n$ indices, and the contribution of $e-e$ interaction is determined as follows:

$$\Delta_{n,n',i,j}(\vec{k}) = \frac{\langle 0| A_{n_i,n_j,i,j}^+(\vec{k}) H_{int} A_{n',n'_{i'},i',j'}^+(\vec{k}) |0\rangle}{\langle 0| A_{n_i,n_j,i,j}^+(\vec{k}) A_{n',n'_{i'},i',j'}^+(\vec{k}) |0\rangle},$$

where $H_{int}$ is the two-particle operator for $e-e$ interaction:

$$H_{int} = \frac{1}{2} \sum_{i_1...i_4} \int \frac{d^2q}{(2\pi)^2} V_{n_i,n_j,n_{i'},n_{j'}}(\vec{q}) A_{n_i,n_j,i,j}^+(\vec{q}) A_{n_{i'},n_{j'},i',j'}^+(\vec{q}) A_{n_i,n_j,i,j}(\vec{q}) A_{n_{i'},n_{j'},i',j'}(\vec{q}).$$

Here, matrix element $V_{n_i,n_j,n_{i'},n_{j'}}(\vec{q})$ is expressed via the Fourier transform of Coulomb Green function $\tilde{D}(\vec{q}, z, z')$ that describes the interaction between the charges at points $z$ and $z'$. The reduction of the Coulomb interaction caused by the finite QW width is taken into account by solving the electrostatic problem in a three-layer system with different dielectric constant values in the QW and in the barriers. We note that the multicomponent nature of single-electron wave functions is allowed for the calculation of $\tilde{V}_{n_{i_1},n_{i_2},n_{j_1},n_{j_2}}(\vec{q})$. The explicit expressions for $\tilde{D}(\vec{q}, z, z')$ and $\tilde{V}_{n_{i_1},n_{i_2},n_{j_1},n_{j_2}}(\vec{q})$ are given in Ref. 19.

According to GGM, the averaging over the ground state in the calculation of $\tilde{E}_{n,n',n_i,n_j}(\vec{k})$ is performed by using

$$(0| A_{n,n',i,j}^+(\vec{q}) A_{n,n',i,j}(\vec{q}) |0) = \nu_{n}^{(i)}(\vec{q})^2 - 1,$$

$s_{n}^{(i)}(\vec{q}) - 1 = \tilde{h}_{n}^{(i)}(\vec{q}) + 2\pi N_{\phi}^{(i)}(\vec{q}).$

Here, $\tilde{h}_{n}^{(i)}(\vec{q})$ is a static structure for the electrons in LL $(n, i)$, $N_{\phi} = L^2/2\pi a_B^2$ (where $L \times L$ is a square of 2D system), $\nu_{n}^{(i)}$ is filling factor of given LL $(n, i)$, and $\tilde{h}_{n}^{(i)}(\vec{q})$ is defined by the Fourier component of the density-density correlation function:

$$\tilde{h}_{n}^{(i)}(\vec{q}) = \nu_{n}^{(i)} \int d^2\vec{r} |g_{n}^{(i)}(\vec{r}) - 1|^2 e^{-i\vec{q}\cdot\vec{r}},$$

in which $g_{n}^{(i)}(\vec{r})$ satisfies the normalization condition

$$\nu_{n}^{(i)} \int d^2\vec{r} |g_{n}^{(i)}(\vec{r}) - 1|^2 = -1.$$

Note that the exact equation $\nu_{n}^{(i)}$ satisfies the normalization condition $\nu_{n}^{(i)} = 1 - 1/M$ as well.

We are now going to illustrate this approach in specific case $3 < \nu < 4$ which corresponds to Fig. 1 with $n_0 = 1$. In the present case, the basis states, from which the relevant magnetoplasmon excitations are constructed, are

$$|1\rangle = A_{0}^{(i) -1,b,b}(\vec{k}),$$

$$|2\rangle = A_{0}^{(i) 1,a,a}(\vec{k}),$$

$$|3\rangle = A_{0}^{(i) 0,0,b,b}(\vec{k}).$$

![FIG. 1. Schematic LL diagram showing the different excitonic transitions between different LLs for 2f + 1 < \nu < 2f + 2, f = 1, 2, ... The full horizontal lines mimic the filling of the LLs. The blue notations label LLs in “single-band” approximation with energy-independent m* and g*, while the black ones are for classification inherent in the eight-band k \cdot p Hamiltonian.](image-url)
By using Eq. (7) in the evaluation of averages, containing two operators $A_{n_{i},j_{i},l_{i}}^{\dagger}(\hat{k})$ over the ground states, and keeping the terms, corresponding to considered basis states, one arrives at the following effective Hamiltonian for zero-momentum magnetoplasmon excitations:

$$H_{MP}(0) = \hat{H}_{HFA} + \delta\hat{H}_{GSMA},$$

(9)

in which correction $\delta\hat{H}_{GSMA}$, arising due to electron correlations, is separated explicitly. In Eq. (9), the term $\hat{H}_{HFA}$ is a so-called mean-field contribution, calculated in the Hartree-Fock approximation\(^{7,8}\) (HFA):

$$\hat{H}_{HFA} = \begin{pmatrix}
    \hat{h}_{11} & (1 - \varepsilon)\hat{h}_{12} & (1 - \varepsilon)\hat{h}_{13} \\
    -(1 - \varepsilon)\hat{h}_{12} & \hat{h}_{22} & -\hat{h}_{23} \\
    -\hat{h}_{13} & -\hat{h}_{23} & \hat{h}_{33}
\end{pmatrix},$$

where $\varepsilon = \nu - 3$ is the fractional part of the filling factor, $\hat{h}_{11}$, $\hat{h}_{22}$, and $\hat{h}_{33}$ are the “single-particle” CR energies, calculated by taking into consideration the terms $V_{HMP}$ and $\varphi_{e-e}$ in single-electron Hamiltonian\(^{21,23}\) $\hat{H}(\varepsilon)$, and the matrix elements are defined as

$$h_{11} = \Sigma_{0}^{(b)} - \Sigma_{-1}^{(b)} - (1 - \varepsilon)\Sigma_{-1,0,0,-1}^{(b,b,b,b)},$$

$$h_{22} = \Sigma_{1}^{(a)} - \Sigma_{1}^{(a)} - \Sigma_{2,2,1}^{(a,a,a,a)}(0),$$

$$h_{33} = \Sigma_{1}^{(b)} - \Sigma_{0}^{(b)} - \Sigma_{0,1,1,0}^{(b,b,b,b)}(0),$$

$$h_{12} = \Sigma_{1}^{(b,a,a)}(0),$$

$$h_{13} = \Sigma_{1}^{(b,b,b)}(0),$$

$$h_{23} = \Sigma_{1}^{(a,a,b)}(0).$$

Here, matrix elements of $e-e$ interaction $E_{n_{j},l_{j},i_{j}}^{(i_{j},j_{j},i_{j})}(\hat{k})$ and exchange corrections to LL energy $\Sigma_{n_{i}}^{(i)}$ have the form

$$E_{n_{j},l_{j},i_{j}}^{(i_{j},j_{j},i_{j})}(\hat{k}) = \int \frac{d^{2}\vec{q}}{(2\pi)^{2}} V_{n_{j},l_{j},i_{j}}^{(i_{j},j_{j},i_{j})}(\vec{q})e^{i\vec{q}\cdot \vec{k}},$$

$$\Sigma_{n_{i}}^{(i)} = -\sum_{n_{i}'}\Sigma_{n_{i},n_{i}'}^{(i,i,i)}(0).$$

(10)

The GSMA correction to the energy of CR modes is written as follows:

$$\delta\hat{H}_{GSMA} = \begin{pmatrix}
    \delta h_{11} & -\delta h_{12} & -\delta h_{13} \\
    0 & \delta h_{22} & 0 \\
    \delta h_{13} & \delta h_{32} & \delta h_{33}
\end{pmatrix},$$

(11)

where

$$\delta h_{11} = \int \frac{d^{2}\vec{q}}{(2\pi)^{2}} \left( \varepsilon + \hat{h}_{e}(\vec{q}) \right)$$

$$\times \left[ V_{b,b,b,b}(\vec{q}) - V_{b,b,b,b}(\vec{q}) - V_{b,b,b,b}(\vec{q}) - V_{b,b,b,b}(\vec{q}) \right],$$

$$\delta h_{22} = \int \frac{d^{2}\vec{q}}{(2\pi)^{2}} \left( \varepsilon + \hat{h}_{e}(\vec{q}) \right)$$

$$\times \left[ V_{b,b,b,b}(\vec{q}) - V_{b,b,b,b}(\vec{q}) - V_{b,b,b,b}(\vec{q}) - V_{b,b,b,b}(\vec{q}) \right],$$

Here, $\hat{h}_{e}(\vec{q})$ is the pair-distribution function in reciprocal space for the correlated electrons in the partially filled LL. Note that $\delta\hat{H}_{GSMA} = 0$ at $\varepsilon = 0$ or 1.

Transition to the model similar to the one, developed by MacDonald and Kallin\(^{8}\) for the case of $2 < \nu < 3$, is performed by the replacement of the eight-component wave function in Eqs. (1) and (2) by the wave function in the parabolic subband\(^{21,23}\)

$$\Psi_{n_{i},k}(\vec{r},z) \rightarrow \tilde{\Psi}(\vec{r},z)\sigma_{z}[n,\hat{k}],$$

(12)

where $\sigma_{z}$ is the Pauli matrix. When changing the LL indices from $(n_{i} - 1)$, $b$, and $(n_{i}, a)$ for $(n_{i}, 1)$ and $(n_{i}, 1)$, one should consider the fact that the g-factor in InAs/AISb QW is negative.

Performing the replacement according to Eq. (12), employing the transformation $\hat{H}_{MP}(0) = \hat{M}^{-1}\hat{H}_{MP}\hat{M}$, where $\hat{M}$ is the diagonal matrix such that $M_{11} = \sqrt{1 - \varepsilon}$, $M_{22} = 1$ and $M_{33} = \sqrt{\varepsilon}$, and taking into consideration the particle-hole symmetry (as in Ref. 5) after some tedious calculations, $\hat{H}_{MP}(0)$ in Eq. (9) is reduced to

$$\hat{H}_{MP}(0) = \begin{pmatrix}
    \hat{h}_{0,1} + 2\varepsilon & 0 & -I\sqrt{2}\varepsilon(1 - \varepsilon) \\
    0 & \hat{h}_{0,2} & 0 \\
    -I\sqrt{2}\varepsilon(1 - \varepsilon) & 0 & \hat{h}_{0,3} + I(1 - \varepsilon)
\end{pmatrix},$$

(13)

where $I$ reflects exchange and correlations within the partially filled LL:

$$I = \frac{1}{2a_{B}} \int \frac{d^{2}\vec{q}}{(2\pi)^{2}} \frac{V(\vec{q})}{1 + \hat{h}_{e}(\vec{q})} \left| \vec{e}(\vec{z}) \right|^{2} \left| \vec{e}(\vec{z}) \right|^{2}. $$

Here, $V(\vec{q})$ is the Fourier component of Coulomb potential, averaged over electron motion along the $z$ axis:

$$V(\vec{q}) = \int dz \left| d\vec{\mathcal{D}}(q_{z},z) \right|^{2} \left| \varepsilon(z) \right|^{2}. $$

In the strict 2D limit $\nu = 2\pi\varepsilon^{2}/q \varepsilon$, where $\varepsilon$ is the static dielectric constant. One can see that $\hat{H}_{MP}(0)$ for $3 < \nu < 4$ in Eq. (13) coincides with the Hamiltonian, derived by MacDonald and Kallin for $2 < \nu < 3$. Note that Hamiltonian in Eq. (9) is more general and does not require the particle-hole symmetry, employed in derivation of Eq. (13).
As it is mentioned above, analytical form of \( \hat{h}_I(q) \) at \( \nu > 3 \) is known only for incompressible quantum-liquid states\(^{48,49}\) at \( \varepsilon = 1/M \) (\( M \) is an odd integer). The quantum-liquid phases are stable only in the vicinity of \( \varepsilon = 1/M \), while at other fillings, an electron solid or stripe phases are energetically favorable\(^5\) for which \( \hat{h}_I(q) \) is unknown. Therefore, one often considers the matrix elements of \( e-e \) interaction as phenomenological parameters, extracting dependence \( I(\varepsilon) \) by fitting CR data.\(^{13-15,17}\)

Furthermore, we compare calculated CR energies obtained by using Eqs.\( (9) \) and \( (13) \) with known functions \( h_I(q) \), as well as with \( \hat{h}_I(q) = -\varepsilon \) (HFA results). Additionally, for the case of Eq.\( (13) \), we will employ the function \( I(\varepsilon) \), which has been calculated by MacDonald and Kallin\(^5\) with the use of a Jastrow-type trial wave function and a hypernetted chain method for the strict 2D limit.

Fig. 2 presents calculation results of two lowest CR energies in symmetric InAs/AlSb QW of 15 nm width with 2DEG concentration of \( n_S = 7.3 \times 10^{11} \text{cm}^{-2} \) in the range \( 3 < \nu < 4 \). Black curves are “single-electron” calculations in the Hartree approximation. The dotted blue curves are the HFA results. The GSMA results for \( \varepsilon = 1/M \) and \( \varepsilon = 1 - 1/M \) are given by open symbols. The left panel conforms to results of diagonalization of \( H_{\text{MP}}(0) \) defined by Eq.\( (9) \), which includes effect of nonparabolicity in the calculations of matrix elements of \( e-e \) interaction. The right panel presents the results, obtained within the “simplified” Hamiltonian in Eq.\( (13) \). The red dotted curve in the right panel corresponds to the calculations with the use of \( I(\varepsilon) \), obtained by the hypernetted chain method.\(^5\)

An important prediction of both models is that energy, corresponding to the CR transition from completely filled LL, is weakly affected by \( e-e \) interaction, while the CR transitions, involving the partially filled LL, are hybridized. It is seen that HFA and GSMA results, obtained within “simplified” Hamiltonian \( (13) \), are in qualitative agreement with more accurate calculations, performed by using the effective Hamiltonian \( \hat{H}_{\text{MP}}(0) \) in Eq.\( (9) \). Including the nonparabolicity effect in the calculations of matrix elements of \( e-e \) interaction leads to additional negative contribution to the CR energy for transition \( (0, b) \rightarrow (1, b) \) (see Fig.\( 1 \) with \( n_0 = 1 \)), as compared with values obtained within “simplified” Hamiltonian in Eq.\( (13) \).

As it is seen from Fig. 2, at \( \varepsilon < 1/3 \) and \( \varepsilon > 2/3 \), CR values obtained by GSMA are very closed to HFA results. However, according to prediction of MacDonald and Kallin,\(^5\) in the range of \( 1/3 < \varepsilon < 2/3 \), the lowest CR transition shifts toward low energy region, as compared with HFA results, and reaches its minimum value at \( \varepsilon = 1/2 \). At \( \varepsilon = 1/M \) or \( \varepsilon = 1 - 1/M \), red dotted curve in Fig. 2(b) coincides with the GSMA values, obtained for incompressible quantum-liquid states.

In addition to MacDonald and Kallin, calculations of \( I(\varepsilon) \) in the range of \( 1/3 < \varepsilon < 2/3 \) were also performed by Asano and Ando\(^6\) by means of numerical diagonalization in finite-size systems. In accordance with their results, \( I \) weakly depends on \( \varepsilon \) and is close to the values obtained within HFA. One can show that calculations of \( I(1/2) \) with the use of \( g_{\nu}^{(0)}(r) \) parameterizations\(^{52} \) for \( \nu = 1/2 \) for polarized composite Fermi sea and Pfaffian states\(^53\) are also close to Asano and Ando result at \( \varepsilon = 1/2 \). The difference between predictions of Refs. 5 and 6 is likely attributed to different ground states manifested in the simulations.

As electron correlations are sensitive to the type of ground state, the behaviour of CR modes at fractional \( \nu \) should be related to competitions between different phases of 2D system.\(^{51,54,55} \) Thus, CR technique could be used to probe correlation effects at steady electron phase in narrow-gap 2D system. To observe effects of \( e-e \) interaction discussed above, one should deal with high-mobility samples such that disorder effects are small.

III. EXPERIMENT

The samples were grown by molecular beam epitaxy on semi-insulating GaAs (001) substrates with a GaSb buffer layer.\(^{25,33,41,42} \) The growth procedure involved a specially designed sequence of interruptions in the supply of materials, which ensured formation of In–Sb bonds at both heterointerfaces.
and, therefore, a high mobility of 2D electrons (in contrast to Al–As bonds). The samples were nominally undoped and amount of electrons in the QW was supplied from surface donors of the GaSb cap layer. Parameters of the samples are given in Table I.

The CR spectra were measured at $T = 2$ K in Faraday configuration using a superconducting coil ($B$ up to 10 T in LNCMI-G). All the spectra were normalized by the sample transmission at $B = 0$. To prevent the interference effects, the substrates of sample structures were wedged with an angle of $\theta$. Pronounced effect of negative persistent photoconductivity attributed to GaSb cap layer, allows one to tune electron concentration by using blue LED illumination. In order control a 2DEG concentration in CR experiments, magnetoresistance was measured as well.

Fig. 3 contains typical CR spectra in samples Tc38 and Tc40. The features at 218.5 cm$^{-1}$ and 230.5 cm$^{-1}$ are related to IR absorption by TO phonons in InAs and GaSb layers. At energies ranging from 255 cm$^{-1}$ to 300 cm$^{-1}$, corresponding to reststrahlen bands in GaAs substrate, we failed to detect the radiation transmitted through the sample.

Experimental spectra, obtained in Tc40 and Tc38 samples at “dark” electron concentration (before illumination by LED), are given in (a) and (b) panels, respectively. As it is seen, the CR lines split nearby magnetic fields corresponding to odd $\nu$ due to non-equidistant spacing of LLs ($\Delta m^*$-splitting). In the fields corresponding to even $\nu$, both samples exhibit a smaller splitting of the CR lines, which is attributed to the difference of the effective g-factor at different-number LLs ($\Delta \mu_g^*$-splitting). By reducing the electron concentration through application of the LED illumination, we got the shift of CR lines at given values of $B$ towards the low-energy region, which is related to subband nonparabolicity InAs/AlSb QW. The bottom panels in Fig. 3 show the CR spectra for sample Tc38 after influence of the LED illumination, reducing the electron concentration to $6.8 \times 10^{11}$ and $6.6 \times 10^{11}$ cm$^{-2}$ for the case of (c) and (d), respectively.

The general feature, observed in Fig. 3 for both samples, is the triple splitting of CR line for transitions with partially filled LLs. Actually, this is the first distinct observation of the triple CR line splitting that indicates the top quality of

![FIG. 3. CR spectra in sample Tc40 (a) and Tc38 (b)–(d). The numbers against the CR lines are values of the magnetic fields, expressed in Tesla. The top panels correspond to CR, measured in the absence of LED illumination. The (c) and (d) panels are CR measurements with LED-reduced 2DEG concentration of $6.8 \times 10^{11}$ and $6.6 \times 10^{11}$ cm$^{-2}$, respectively. The arrows indicate the fields corresponding to the integer filling factors of LLs.]

<table>
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<tr>
<th>Sample</th>
<th>QW width, nm</th>
<th>$n_s$, $10^{11}$ cm$^{-2}$</th>
<th>$\mu$, $10^3$ cm$^2$/Vs</th>
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<td>7.1</td>
<td>5.4</td>
</tr>
<tr>
<td>Tc40</td>
<td>15</td>
<td>7.3</td>
<td>5.5</td>
</tr>
</tbody>
</table>

TABLE I. Sample parameters at $T = 2$ K.
our samples, in which the LL width seems to be significantly less than in the structures studied so far (cf. Ref. 27). The usage of blue LED illumination allows us to reduce an electron concentration to less than $2.5 \times 10^{11} \text{cm}^{-2}$. However, neither $D_{g}^{*}$—nor $D_{m}^{*}$—splitting of CR lines does not arise in the transmission spectra in this case, that indicates increasing of the disorder ascendancy and reduction of e-e interaction effects.

**IV. RESULTS AND DISCUSSION**

Fig. 4 provides a comparison between experimental data and calculation results, obtained by “single-electron” approximation based on $H_{k_{x}}^{2}$. The red curves conform to CR transitions between LLs $(n, a)$ and $(n + 1, a)$, while the black curves describe the CR transitions for LLs with $i = b$. The panels in Fig. 4 are connected with the ones in Fig. 3. Black symbols are experimental values, extracted from the CR line positions in corresponding spectra. A good agreement between theoretical calculations and experimental values of CR energies allows one to reliably identify the CR transitions observed in transmission spectra.

A noteworthy feature, which is inherent in all CR spectra, is the dependence of CR energies on magnetic field at $3.5 < \nu < 4$. One can see that the energy of transition from the lower LL (short wavelength transition) increases with a rising magnetic field, in conformity with the “single-electron” calculations. However, the energy of transition from the upper, partially filled level is getting progressively lower than “single-electron” value. Two effects shall be discussed which might possibly explain such behaviour of CR transition.

Firstly, it is known that electron-phonon interaction could produce relevant corrections to the CR energies in the range near TO and LO phonon energies of QW materials ($\lambda_{\text{TO}}^{1} = 218.5 \text{cm}^{-1}$ and $\lambda_{\text{LO}}^{1} = 242.5$ for InAs). This is a polaron effect. We note that single-particle electron-phonon interaction is the same for both spin configurations, however, the many-body effects due to the different occupancies of the two spin states lead to very different polaron corrections to the CR transitions from spin-split LLs in TO-LO phonon energy region. As a result, the energy of CR transition from partially filled LL bends to low-energy region, when CR energy tends to $\lambda_{\text{TO}}^{1}$ (see Fig. 1 in Ref. 16).

**FIG. 4.** “Single-electron” CR energies as functions of magnetic fields for Tc40 (a) and Tc38 (b)-(d) samples. The symbols are experimental values, obtained from the lines in corresponding CR spectra. The arrows indicate magnetic fields corresponding to integer $\nu$. The insets in the panels (a)-(c) show the CR energies in the range of $3.5 < \nu < 4$. 

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A second effect that might explain the observed behaviour of CR transitions in our experiments has optical origin. The optical response of the system is governed by two strongly resonant processes, the resonant dielectric function of the material at \( \lambda_{TO}^{-1} \) (even if the polaron effect is small) and the CR absorption of 2D electron system, where the latter shifts with increasing magnetic field.\(^{56}\) Thus, both effects might be observed at the energies close to \( \lambda_{TO}^{-1} = 218.5 \text{ cm}^{-1} \).

It is seen from Figs. 3(a) and 3(b) and Figs. 4(a) and 4(b) that the maximum shift of low-energy peaks in CR spectra is indeed observed in the vicinity of \( \lambda_{TO}^{-1} \). However, this specific behaviour is also reproduced at lower electron concentrations at the energies far from \( \lambda_{TO}^{-1} \) (see Fig. 4(d)). This indicates that observed behaviour cannot be related to the above mentioned polaron effects.

As it is shown in Sec. III, the behaviour of CR modes in the range of \( 3.5 < \nu < 4 \) can be attributed to \( e-e \) interaction effect. To describe this effect in both samples, we start from effective Hamiltonian \( \hat{H}_{MP}(0) \) in Eq. (9), obtained for zero-energy magnetoplasmon excitations. Since the ground state in our samples, defining the pair-distribution function at given \( \nu \) and, respectively, the behaviour of CR modes, is unknown, we have to introduce a phenomenological parameter \( \Delta_e \), which describes correlation contribution to CR energies:

\[
\int \frac{d^2 q}{(2\pi)^2} \left( \epsilon + \tilde{h}_e(q) \right) \mathcal{V}^{(i_1,j_1,i_2,j_2)}_{n_1,n_2,n_3,n_4}(q) \approx \Delta_e \hat{E}^{(i_1,j_1,i_2,j_2)}_{n_1,n_2,n_3,n_4}(0).
\]

As a result, correction \( \delta \hat{H}_{\text{GSMA}} \) to HFA contribution in Eq. (13) is written in the form

\[
\delta \hat{H}_{\text{GSMA}} \approx \Delta_e \begin{pmatrix}
\delta \hat{h}_{11} & -\delta \hat{h}_{12} & -\delta \hat{h}_{13} \\
0 & \delta \hat{h}_{22} & 0 \\
\delta \hat{h}_{13} & \delta \hat{h}_{32} & \delta \hat{h}_{33}
\end{pmatrix},
\]

(14)

where

\[
\delta \hat{h}_{11} = \hat{E}^{(b,b,b)}_{0,0,0,0}(0) - \hat{E}^{(b,b,b)}_{1,0,0,0}(0) - \hat{E}^{(b,b,b)}_{0,1,0,0}(0),
\]

\[
\delta \hat{h}_{12} = \hat{E}^{(b,a,b)}_{0,0,2,0}(0) - \hat{E}^{(a,b,b)}_{0,1,0,0}(0),
\]

\[
\delta \hat{h}_{13} = \hat{E}^{(b,b,b)}_{1,0,1,0}(0) - \hat{E}^{(b,b,b)}_{0,0,0,0}(0) + \hat{E}^{(b,b,b)}_{0,1,1,0}(0),
\]

\[
\delta \hat{h}_{22} = \hat{E}^{(b,b,b)}_{0,0,0,0}(0) - \hat{E}^{(a,b,a)}_{0,0,0,0}(0),
\]

\[
\delta \hat{h}_{33} = \hat{E}^{(b,b,b)}_{1,1,0,0}(0).
\]

Note that \( \tilde{h}_e(q) = -\epsilon \) is exact pair-distribution function at integer filling factors, therefore \( \Delta_e = 0 \) at \( \epsilon = 0 \) or 1.

Using Eq. (9) with \( \delta \hat{H}_{\text{GSMA}} \) in the form (14) is more preferable for fitting experimental data because this model more accurately takes into account the nonparabolicity effect on the electron wave functions than Eq. (13). One can also show that current model turns into Hamiltonian in Eq. (13) with phenomenological parameter \( l \) under the transformation, described by Eq. (12).

The values of \( \Delta_e \), obtained by fitting experimental CR energies in both samples in the range of \( 3.5 < \nu < 4 \), are given in Fig. 5. The values in the left panel are obtained for symmetric InAs/AlSb QW profile. The right panel are for the case of asymmetric QW. The black, red, green, and blue symbols are connected with panels (a)–(d) in Fig. 4, respectively. Solid lines correspond to the values of \( \Delta_e \), which are extracted from \( \tilde{h}_e(q) \), calculated by MacDonald and Kallin\(^5\) for strict 2D limit. Note that to obtain \( \Delta_e \) from \( \hat{H}_{MP}(0) \) in Eq. (13), one should employ inverse transformation \( \hat{M}^{-1} \).

It is seen that \( \Delta_e \) in both samples is rapidly decreasing when \( \epsilon \) tends to \( 1/2 \), while in the vicinity of \( \epsilon = 1 \), it is close to HFA results. The values of \( \Delta_e \) are weakly affected by QW profile and QW width and depend on \( n_s \) and magnetic field strength only via dependence of \( \epsilon \). Thus, it is seen that our experimental data on CR in InAs/AlSb QWs at \( T = 2 \text{ K} \) in the range of \( 3 < \nu < 4 \) confirm prediction of CR mode behaviour, performed by MacDonald and Kallin.\(^5\) The similar

![FIG. 5. The \( \epsilon \) dependence of effective parameter \( \overline{\Delta}_c \), describing effects of electron correlations for the case of symmetric (a) and asymmetric (b) InAs/AlSb QW profile. Black and red symbols correspond to the samples Tc40 and Tc38 in the absence of LED illumination. Green and blue symbols conform to CR energies in Tc38 sample with LED reduced electron concentration of \( 6.8 \times 10^{11} \) and \( 6.6 \times 10^{11} \text{ cm}^{-2} \), respectively. The result of MacDonald and Kallin for strict 2D limit is shown by the solid line. The value \( \overline{\Delta}_c = 0 \) corresponds to HFA results.](This article is copyrighted as indicated in the article. Reuse of AIP content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to | IP: 162.38.137.128 On: Thu, 08 Oct 2015 16:20:20)
behaviour of CR, that we observe $3 < \nu < 4$, should also
takes place at $2 < \nu < 3$. However, in this range of $\nu$, CR
energies in our samples correspond to reststrahlen bands in
GaAs substrate (from 250 cm$^{-1}$ to 300 cm$^{-1}$) and AlSb layer
(from 320 cm$^{-1}$ to 330 cm$^{-1}$). In lower magnetic fields, such
that $\nu > 4$, the role of impurities increases, resulting in reduction
of $e$-$e$ interaction effects on CR energies and a good
agreement between experimental values and “single-
electron” calculations.

It should be emphasized that the prediction of Asano
and Ando,\textsuperscript{6} according to which the behaviour of CR modes
agreement between experimental values and “single-
electron” interactions based on eight-band $k \cdot p$ Hamiltonian. In the range $3 < \nu < 4$, the energy of CR transition from the upper partially filled LL is getting progressively lower with a rising magnetic field than the single-electron values, while transition from fully-filled LL demonstrates a behavior in accordance with single-electron picture. The strength of the splitting increases when occupation of the partially filled LL tends to a half, that is, attributed to $e$-$e$ interaction effect. We have shown that such behaviour of CR modes can be quantitatively described if one takes into account both electron correlations, as it is done by MacDonald and Kallin,\textsuperscript{12} and the mixing between conduction and valence bands, described by eight-band $k \cdot p$ Hamiltonian, in the calculations of matrix elements of $e$-$e$ interaction.

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