Investigation on reconstruction methods applied to 3D terahertz computed Tomography

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http://www.cpmoh.cnrs.fr/SLAM/terahertz-spectroscopy-and-imaging/

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I- Introduction

II- 3D Imaging
   1) 3D Reconstruction algorithm
   2) Results: mm Imaging & TDS imaging

III- Conclusions & Perspectives
Problems and limit with T-ray imaging & CT

- In T-ray CT, T-ray propagation can not be correctly described by a geometrical ray line which is valid in X-ray CT,

- Diffraction or scattering effects blur or deform the reconstructed image.

- Complicated signal analysis for complex structure (internal or external shape), the multiple reflections and refractions of the THz radiation.
✓ 3D Modelisation et Reverse Engineering, prototyping

✓ Visualization of 3D objects with THz radiation

Industrial Applications: NDE, defects, cracks, bubbles, composite material defects such as delamination, porosities and inclusions.

Archeology
Anthropology
Art conservation
Security…

Broadband THz-TDS imaging
2D Imaging : results

Some results

Diffraction limited resolution

\[ d_{\text{min}} = 0.61 \frac{\lambda}{N_A} \]

Images at 2 THz

500\(\mu\)m

410\(\mu\)m

270\(\mu\)m

200\(\mu\)m

Investigation of graphite properties (pencil leads)
- THz spectroscopy of graphite
- THz imaging of documents written with pencil leads

H for hardness (more clay, less graphite)
B for blackness (less clay, more graphite).

• Radon’s Theorem (1917)
  Reconstruction

\[ f(x, y) = \int_{\theta=0}^{2\pi} \int_{\rho=-\infty}^{+\infty} R(\theta, \rho) \delta(\rho - x \cos \theta - y \sin \theta) d\theta d\rho \]
Tomography

Back Fourier Transformation

$p(u, \Theta)$

Fourier Transform

$P(\rho, \Theta)$

Filtering

$P(\rho, \Theta)$

inverse Fourier transform

$p'(u, \Theta)$

Reconstructed images

$f(x,y)$

Back projection

Sinograms

Transparent plastic cell

Amplitude image

Delay image

II- 3D Imaging
Advantages & drawbacks

For each pixel, the THz waveform as a function of time. Final tomographic THz images can be obtained maximum value of the THz amplitude signal (amplitude) time delay: average of the different peaks (delay image).

Experimental

Amplitude Phase } Images

Absorption and refraction losses

Trade off: complexity, precision resolution

Time acquisition
Tomography

Spatial discretization

Discretization of the Radon integral

Frequency discretization

Radial discretization induces Irregularity in the interpolation

Information missing
Phantoms reconstructed from CW acquisitions
Acquisition of phantoms made of Foam or Teflon, with hole or metallic bars, 18 projections sized around 100*100.

- Hyper signal at interfaces due to the signal losses,
- Hyper signal at contours in reconstructed images
- Non uniform signal loss near the interfaces.
- T Ray attenuation negligible inside the object?
- T Ray attenuation non linear to the density of traversed matter (compared to X Ray)?

**Foam (n=1.05)**

Number of projections
Foam cube and metallic bar (240 GHz)

- step : 90° 2 projections
- step : 50° 4 projections
- step : 30° 6 projections
- step : 20° 9 projections
- step : 10° 18 projections
3D computed T

Time domain approach

Difficult multi-peak analysis: modeling using a ray-tracing software, taking into account the shape of the sample, the characteristics of the THz beam (size, divergence, broadband spectral range) and the numerical aperture of the collecting lens before the detector.

TDs: multi-peak averaging technique

Ten millimeters diameter Teflon cylinder with off-axis 3.4 mm cylindrical hole: (a) amplitude sinogram from the THz main pulse amplitude, (b) time sinogram from the THz main pulse time delay, (c) time sinogram from the multi-peak averaging, and (d) time sinogram from theoretical values
backprojection algorithm is able to properly reconstruct the shape of the target with a good precision.
Inverse Radon Transform : Back fourier Transform ( BFP)

The reconstruction process : given by inverse Radon transform : recovers the original domain from the projections.

Given a sinogram $S$ containing several $\mathcal{R}(\rho)$ values ($\theta \in [0,\pi]$ and $\rho \in \mathbb{R}$), the discrete inverse Radon transform computes each image pixel

$$I(i,j) = \sum_{i_{\theta}=0}^{N_{\theta}-1} \sum_{\rho=-\frac{N_{\rho}}{2}}^{\frac{N_{\rho}}{2}} W_{\theta}(i_{\theta})(\rho) A(\theta,\rho),(i,j)$$

where:

- $\theta(i_{\theta}) = i_{\theta} \frac{\pi}{N_{\theta}}$,

- $A(\theta,\rho),(i,j)$ is the weight-matrix defining the weight value between each pixel and each projection line,

- $W_{\theta}(\rho) = \sum_{v=-\frac{N_{\rho}}{2}}^{\frac{N_{\rho}}{2}} |v| \left( \sum_{\rho_{s}=-\frac{N_{\rho}}{2}}^{\frac{N_{\rho}}{2}} \mathcal{R}(\rho_{s}) e^{-i2\pi \rho_{s} v} \right) e^{i2\pi \rho v}$. filtered projections

the ramp filter $|v|

BFP is very sensitive to the projection number $N_{\theta}$.

**BFP : noise sensitivity, artifact, blurring, streak**
Tomographic reconstruction method

Iterative method: SART (Simultaneous algebraic Reconstruction Technique)
OSEM (Ordered Subsets Expectation Method)

the idea

- make initial guess
- check how well it corresponds to the measured data [back-projection]
- calculate the difference between the result and real measurement (Compute direct radon transform of the image)
- correct the values
- repeat until results satisfying
Tomographic reconstruction method
iterative process

simple example

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<th>3</th>
<th>12</th>
<th>3</th>
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<tbody>
<tr>
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<td>3</td>
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<tr>
<td>3</td>
<td>12</td>
<td>12</td>
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</tbody>
</table>

- make initial guess
- while convergence not reached // iterations
  - for each projection
    - for each ray
      - compute back-projection
      - compute difference to measured projection
      - distribute difference
    - end for
  - end for
- end while

the equation system

\[
W \cdot f = p
\]

\[
M = \#\text{projections} \times \#\text{rays}
\]

\[
W - \text{the weight matrix}
\]

\[
\text{what pixels are influenced by one ray}
\]

\[
N = \text{image size}^2 = \#\text{pixels}
\]

the kaczmarz method

\[
I^{k+1}(i,j) = I^k(i,j) + \lambda \frac{\sum_{\rho=0}^{N_\theta-1} A(\theta,\rho),(i,j) }{ \sum_{\rho=0}^{N_\theta-1} A(\theta,\rho),(i,j) } \left( \frac{ R_{\theta}(\rho) - R^k_{\theta}(\rho) }{ D_{\theta}(\rho) } \right)
\]

\[
W = \sum_{i=0}^{N_\theta-1} \sum_{\rho=0}^{N_\rho-1} A(\theta,\rho),(i,j) \frac{ R_{\theta}(\rho) }{ R^k_{\theta}(\rho) } 
\]

where \( D_{\theta}(\rho) = \sum_{i=0}^{N_\theta-1} \sum_{j=0}^{N_j-1} A(\theta,\rho),(i,j) \) is the norm of the segment \((\theta,\rho)\) crossing the image:

SART

OSEM

error correction is multiplicative

II- 3D Imaging

OSEM method, the convergence of the solution is longer than with the SART
Sinograms of two metallic bars (12mm diameter) separated by 50 mm, with a projection number $N_q = 72$.

(a) Ideal theoretical sinogram calculated by direct Radon transformation of a synthetic model.

(b) Acquisition using the millimeter wave tomographic scanner with the 110 GHz source.

Cross sections of two metallic bars separated by 50 mm.

(a) Ideal synthetic cross section of the sample.

(b) BFP reconstruction

(c) SART reconstruction

(d) OSEM

Comparison of iterative method and BSP

<table>
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<tr>
<th>ART</th>
<th>FBP [filtered back projection]</th>
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<tbody>
<tr>
<td>+ better noise tolerance</td>
<td>+ more computationaly efficient</td>
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<tr>
<td>+ needs less projections</td>
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<tr>
<td>+ better handling of non-uniformly distributed projection datasets</td>
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White foam parallelepiped (30mm cube size) drilled by two oblique metallic bars (6mm diameter).
(a) Photograph of the 3D sample.
(b) BFP reconstruction.
(c) ART reconstruction.
(d) OSEM reconstruction.

Acquisition with the 240 GHz source.
**Video 1**

**Vidéos 2**
With a millimeter wave source associated with an infrared temperature sensor, we investigated the reconstruction methods applied to THz CT. The system is easy-to-align, low-cost and portable.

Comparison with the commonly used BFP reconstruction algorithm, we pointed out the ability of the SART and OSEM algorithms to properly reconstruct cross-sectional images.

The SART method applied to THz CT obtains equivalent accuracy and quality than BFP. For a limited number of projections (less than 25), we noticed a quantitative degradation of the BFP reconstruction whereas SART and OSEM methods can already offer an optimized reconstruction quality. Consequently, iterative methods allow to reduce the acquisition time (i.e. the projection number) without noticeable quality and accuracy losses.

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