Carrier distribution and population inversion in Landau level system of cascade GaAs/AlAs quantum well structures in strong tilted magnetic field

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Outline:

• population inversion in Landau level system in resonant tunneling quantum well structures
• intersubband transitions matrix elements in tilted magnetic field and optical gain estimation
• conclusions
Motivation:

- Wide range tunable quantum cascade THz laser based on transitions between Landau levels of different subbands in resonant tunneling quantum well structures
\( \hbar \omega = \Delta E_{21} - \hbar \omega_C \)
Scattering between Landau levels?

- optical phonon scattering is excluded since intersubband spacing is lower than the optical phonon energy
- acoustical phonon and impurity scattering are strongly suppressed

Electron-electron scattering is the most important relaxation mechanism determining the population of the Landau levels

The calculation of the e-e scattering rates for both intersubband and intrasubband scattering is the main point of the present work
\[
\frac{1}{\tau_{(i,j)\rightarrow(f,g)}} = \frac{2}{L^2\alpha} \sum_{k_i} \frac{1}{\tilde{\tau}_{(i,j)\rightarrow(f,g)}(k_i)} \\
\frac{1}{\tilde{\tau}_{(i,j)\rightarrow(f,g)}(k_i)} = \frac{2\pi}{\hbar} \sum_{k_j,k_f,k_g} |V_{(i,j)(f,g)}(k_i,k_j,k_f,k_g)|^2 \cdot \frac{N_j}{\alpha} \left[ 1 - \frac{N_f}{\alpha} \right] \left[ 1 - \frac{N_g}{\alpha} \right] \cdot F_{(i,j)(f,g)}(E_i + E_j - E_f - E_g)
\]

\[
V_{(i,j)(f,g)}(k_i,k_j,k_f,k_g) = \int dr_1 dr_2 \psi^*_{f,k_f}(r_1)\psi_{i,k_i}(r_1) \frac{e^2}{\varepsilon_s |r_1 - r_2|} \psi^*_{g,k_g}(r_2)\psi_{j,k_j}(r_2)
\]

\[
V_{(i,j)(f,g)}(k_i,k_j,k_f,k_g) = \frac{2e^2}{\varepsilon_s L} \int dy_1 dy_2 dz_1 dz_2 \Phi^*_{m_f}(y_1 - k_f\ell^2)\Phi^*_{m_j}(y_1 - k_i\ell^2) \Phi^*_{m_g}(y_2 - k_g\ell^2)\Phi^*_{m_m}(y_2 - k_f\ell^2) \times \\
\times \phi_{(v,f,m_f)}(z_1)\phi_{(v,i,m_i)}(z_1) \phi_{(v,g,m_g)}(z_2)\phi_{(v,j,m_m)}(z_2) \cdot K_0 \left( k_j - k_f \sqrt{(y_1 - y_2)^2 + (z_1 - z_2)^2} \right) \cdot \delta_{k_i + k_j, k_f + k_g}
\]

\[K_0(x) - \text{modified 0-th order Bessel function of the second kind (Macdonald function)}\]

\[
F_{(i,j)(f,g)}(E) = \frac{1}{\pi} \cdot \frac{\Gamma}{E^2 + \Gamma^2}
\]

\[\Gamma = 4\Gamma_0\]

\[\Gamma_0 - \text{halfwidth of the Landau level}\]
GaAs$_{0.3}$Ga$_{0.7}$As with the well width of 25 nm

$N_{(2,0)} = 10^9$ cm$^{-2}$, $N_{(1,1)} = 10^9$ cm$^{-2}$, $N_{(1,0)} = 10^9$ cm$^{-2}$  \hspace{1cm} \omega_C \tau \gg 1
\( \hbar \omega = \Delta E_{21} - \hbar \omega_C \)

GaAs/Al\(_{0.3}\)Ga\(_{0.7}\)As

Well/barrier width
25/10 nm

\( \Delta E_{12} = 20.4 \text{ meV} \)

\( B = 5 - 10 \text{ T} \)
\( \hbar \omega = 11.8 - 3.2 \text{ meV} \)
\( \nu = 95 - 26 \text{ cm}^{-1} \)
\( \lambda = 105 - 390 \text{ µm} \)
\( f = 2.86 - 0.77 \text{ THz} \)
B=7 T

\[ N_{(i,j)}, 10^6 \text{ cm}^{-2} \]

\[ N_d, 10^9 \text{ cm}^{-2} \]

(2,0)

(1,2)

(1,3)

(1,1)

(1,4)

(2,1)
But – the transitions of interest are optically forbidden, when magnetic field is normal to the structure layers!!!

\[
\psi_{(v,n,k_x)}(x, y, z) = \exp(ik_x x) \cdot \varphi_v(z) \Phi_n(y - k_x \ell^2) 
\]

\[
D_{(2,0)\to(1,1)} = \left\langle \psi_{(2,0,k_1)}(r) \bigg| xe_x + ye_y + ze_z \bigg| \psi_{(1,1,k_2)}(r) \right\rangle = \\
\begin{align*}
= & \langle \exp(ik_1 x) | x | \exp(ik_1 x) \rangle \langle \varphi_2(z) | \varphi_1(z) \rangle \cdot \langle \Phi_0(y) | \Phi_1(y) \rangle \cdot e_x + \\
& + \delta_{k_1,k_2} \cdot \langle \varphi_2(z) | \varphi_1(z) \rangle \cdot \langle \Phi_0(y) | y \Phi_1(y) \rangle \cdot e_y + \\
& + \delta_{k_1,k_2} \cdot \langle \varphi_2(z) | z \varphi_1(z) \rangle \cdot \langle \Phi_0(y) | \Phi_2(y) \rangle \cdot e_z \\
\end{align*}
\]

\[
\langle \varphi_2(z) | \varphi_1(z) \rangle = 0 \quad \Rightarrow \quad D_{(2,0)\to(1,1)} = 0
\]

\[
\langle \Phi_0(y) | \Phi_1(y) \rangle = 0
\]
Tilted magnetic field

If $\hbar \omega_c \leq \Delta E_{12}$

$$\psi(x, y, z) \approx \exp(ik_x x) \cdot \varphi_{\nu}(z) \cdot \Phi_n \left( y - k_x \ell^2_{\perp} - \langle z \rangle_{\nu} \frac{\ell^2_{\perp}}{\ell^2_{\parallel}} \right)$$

$\varphi_{\nu}(z)$ - wavefunction of the $\nu$ - th subband

$$\langle z \rangle_{\nu} = \int dz \; z \cdot |\varphi_{\nu}(z)|^2$$ - average value of the $z$ - coordinate in the $\nu$ - th subband

$\ell_{\perp} = \sqrt{\frac{\hbar c}{eB_{\perp}}}$ and $\ell_{\parallel} = \sqrt{\frac{\hbar c}{eB_{\parallel}}}$ - magnetic lengths of components $B_{\perp}$ and $B_{\parallel}$

$$|D_{(2,0)\rightarrow(1,1)}|^2 = |D_0|^2 \cdot |D_{\text{mix}}|^2$$

$$|D_0|^2 = |\langle \varphi_2(z) \rangle \varphi_1(z)|^2$$

$$|D_{\text{mix}}|^2 = \frac{\xi^2}{2} \cdot \exp \left( -\frac{\xi^2}{2} \right)$$

$$\xi = \left[ \langle z \rangle_2 - \langle z \rangle_1 \right] \cdot \frac{\ell_{\perp}}{\ell^2_{\parallel}}$$
$B_\perp = 5\text{T}$
Asymmetric double well structure

\[ a_1 = 25 \text{ nm} \]
\[ b = 2 \text{ nm} \]
GaAs\Al_{0.3}Ga_{0.7}As \quad a_L=250 \text{ Å} \quad b=20 \text{ Å}

Optical gain – up to 30 cm\text{-1} for 50 periods structure
Conclusions

• a resonant tunneling quantum cascade structure was proposed to achieve population inversion and tunable stimulated emission in Landay level system
• inter-Landau level electron-electron scattering times were calculated for both intersubband and intrasubband scattering transitions and the possibility of population inversion in wide range of magnetic fields was demonstrated
• a tilted magnetic field as well as asymmetric double-well active element construction allows to achieve considerable values of transition matrix elements and optical gain
• proposed mechanism allows to achieve a wide range tuning of the emission frequency
Thank you for the attention